Inflation Dynamics with Bounded Rationality

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Abstract

There is an unbalanced specification in the standard new Keynesian model. In the model, stickiness is assumed in the price setting, and then an individual firm has a fixed probability to change its price in any given period, which means that the market is imperfect. On the other hand, an individual firm is assumed to be one that can conduct profit maximization and calculate the degree of nominal rigidity in the future completely. In order to avoid this unbalanced specification, we suppose that firms choose the price with bounded rationality. Concretely, we assume that firms refer to lagged inflation in the price setting, which is one of the simplest forms to express bounded rationality. We then obtain the hybrid new Keynesian Phillips curve to express inflation dynamics, named the sticky price with bounded rationality Phillips curve (SPBR).

1. Introduction & Empirical Literature

During the past two decades, significant improvements have been made in theoretical and empirical analysis, relating to both price setting and persistent inflation mechanisms. Although many studies have been developed in terms of inflation models which are even used to formulate and evaluate actual monetary policies, a number of questions have yet to be answered on this topic. As Woodford (2007) stated, "Some argue that these two desiderate
remain in considerable tension with one another—that one cannot insist upon optimizing foundations, at least under the current state of knowledge, without substantial sacrifice of quantitative realism."

One critical topic regarding the gap between theoretical deduction and empirical practice is how to treat lags and leads of inflation and output on the actual one. There are no lagged terms in the standard new Keynesian model or new Keynesian Phillips Curve (NKPC). However, a model with lagged terms, called the hybrid new Keynesian model which is short for microeconomic foundation, is being utilized, as well as by economists, businessmen and econometric researchers, in macro econometric models of central banks and international financial institutions because of its good performance on real data.

The history of economics reveals that a sensible principle is practical rather than theoretical. Fuhrer (1997) emphasizes the importance of backward-looking behavior in price specifications in the Taylor’s contract model, and concludes that a mixed backward-looking/forward-looking price specification provides more reasonable behavior on empirical performance. It is well known that there is a strong correlation between actual inflation and lags (e.g., Mankiw, 2001; Gordon, 1997). Fuhrer and Moore (1995) coin a new technical term, “inflation persistence”, to explain this phenomenon. To resolve the puzzle of inflation persistence, several preceding theoretical studies have been performed.

Christiano et al. (2005) present a hybrid new Keynesian Phillips curve to analyze inflation and output persistence with a special assumption. The assumption is that some firms that cannot change their prices do not maintain it, but rather automatically discount prices according to the last period’s inflation rate. However, as Angeloni et al. (2006) pointed out, such an ad hoc assumption as an automatic backward-looking indexation cannot be consistent with macroeconomic evidence. In support of this, Woodford (2007) argues that "the model's (Christiano et al, 2005) implication that individual prices should continuously adjust in response to changes in prices
elsewhere in the economy flies in the face of the survey evidence."

Gali and Gertler (1999) allow for a subset of firms using a backward-looking rule to set prices to obtain the hybrid new Keynesian Phillips curve. They maintain that the hybrid new Keynesian Phillips curve is more persuasive than the New Keynesian Phillips curve in terms of fitting actual data. Their work, however, is exposed to incisive criticism of both their theoretical approach and empirical method. Although lags of inflation are crucial to explain the current one, the simple rule-of-thumb behavior assumption omits theoretical rationality. In fact, it is irrational to assert that firms set their prices by the backward-looking rule. From the perspective of firms profit maximization or cost minimization, menu cost will be at least saved by maintaining prices, rather than changing them.

Mankiw and Reis (2002) propose another mechanism. They examine a dynamic price model based on an assumption that information disseminates slowly throughout the population, i.e., "sticky-information" compared to "sticky-price". This assumption allows the model to obtain a lagged inflation term in NKPC. They contend that the maximum effect of monetary policy shocks will appear several periods later. However, as Dupor and Tsuruga (2005) claimed, the result of this sticky-information model depends largely on the diffusion structure of information. Thus, in the case in which the diffusion structure of information is assumed in a fixed duration, such as Taylor (1979), the persistent and hump-shaped inflation and output dynamics will disappear.

This paper proposes a new model to explain inflation dynamics. The essence of the model is that firms choose the price with bounded rationality. In the standard NKPC model based on Calvo (1983), the aggregate price index is a weighted average of the price charged and not-charged by firms, which means that there is a nominal rigidity of price, which is one of the most important features of the NKPC model. On the other hand, firms choose

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an optimal price based on monopolistic competition, which implies that the firms are implicitly assumed to be market-clearing ones. It is unbalanced for an individual firm to be treated as one that can conduct profit maximization and calculate the degree of nominal rigidity in the future completely, while sticky-price is also assumed in the model simultaneously.

Blanchard and Gali (2007) introduce real wage rigidities -one of bounded rationality of the market- in the model. They then derive a simple representation of inflation as a function of lagged and expected inflation, the unemployment rate, and the change in the price of non-produced inputs. Although their model’s specification differs greatly from that of our model, the concept of its hypothesis is similar to ours.

Mishkin (2007) argues that the recent changes in inflation dynamics are less persistent and more likely to gravitate to a trend level, and expectations have become better anchored. Bernanke (2010) and Donald Kohn (2010) hold a similar view. Ball and Mazumdar (2011), adding the hypothesis of anchored inflation expectations into the traditional Phillips curve, examine inflation dynamics in the U.S. since 1960 and forecast the inflation rate during the Great Recession. Although they examine the traditional Phillips curve, they provide us with a number of discerning ideas. For example, the fact that the hypothesis of anchored expectations can refine the prediction of inflation dynamics suggests that firms are not perfect market-clearing ones. We add expectations with bounded rationality into the model to obtain the hybrid new Keynesian Phillips curve.

In the next section, we present our model in detail. We call the model the sticky-price with bounded rationality (SPBR), in contrast with the NKPC. Section II shows the simulation and estimation of the model, and examines inflation dynamic properties. Section III analyzees bounded rationality

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2 For refining the prediction of inflation dynamics, Ball and Mazumdar (2011) also modify two points of Phillips curve: 1) measuring core inflation with the weighted median of consumer price inflation rate across industries; and 2) allowing the slope of Phillips curve to change with the level of variance of inflation.
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cmpared to sticky price, and explains why bounded rationality needs to be
introduced in the model. We conclude our paper in section IV.

2. Sticky Price with Bounded Rationality Model

We will directly introduce the key aggregate relationships, rather than
working through the details of the derivation. Concerning the firm’s desired
price at period, the price would maximize profit. The desired price can be
presented as follows with all variables expressed in logs:

\[ p_t^* = p_t + \alpha y_t \]

(1)

where \( p_t^* \) is the desired price level at time \( t \); \( p_t \) is the overall price
level; and \( y_t \) interprets the output gap for potential output, which is
normalized to zero here. The parameter \( \alpha \) is the degree of the output gap on
the desired price level, and \( \alpha > 0 \). This equation shows that the firm’s
desired price depends on the overall price level and output gap positively,
and that the gap of price level \( p_t^* - p_t \) rises in economic booms and falls in
recessions. Not deriving the equation from a firm’s profit maximization, we
follow our specification, which is consistent with that of Mankiw and Reis
(2002)\(^3\).

In this model, we assume the sticky-price mechanism as Calvo (1983),
i.e., each firm has a fixed probability \( \rho \) in any given period that the firm
may adjust its price during that period and, hence, the probability that the
firm must keep its price unchanged is \( 1 - \rho \), where \( 0 \leq \rho \leq 1 \). This
probability is independent of the time elapsed since the last price revision.
Firms are identical ex ante, except for the differentiated product that they
produce and for their pricing history. We assume that each firm faces a

\(^3\)The baseline specification of this model is in accordance with the standard new Keynesian
model, which assumes that each firm is identical and competes monopolistically following
conventional constant price elasticity of the demand curve for its product. Then, the overall price level becomes as a weighted average of the lagged price level \( p_{t-1} \) and the adjustment price level, \( x_t \), as follows:

\[
p_t = \rho x_t + (1 - \rho) p_{t-1} \tag{2}
\]

This sticky-price mechanism, one of the most important features of the Calvo's model, implies that the market is imperfect. As the standard Calvo model sets, firms do not set the adjustment price level \( x_t \) to be equal to \( p_t^* \), but side up the degree of stickiness of price and then set up a price as a convex combination of the future adjustment price level, \( x_{t+1} \), another of crucial feature of Calvo's model. We can notice a contradiction between these two specifications. The market is assumed to be imperfect, but each firm is assumed to be a market-clearing one. In order to correct this contradiction, we assume that firms are not market-clearing price-setters but rather price-setters with bounded rationality. Considering the evidence of the importance of the lagged inflation as mentioned above, firms set the adjustment price level, \( x_t \), as follows:

\[
x_t = \rho \left( p_t^* + \beta \pi_{t-1} \right) + (1 - \rho) E_t x_{t+1} = \rho E_t \sum_{j=0}^{\infty} (1 - \rho)^j \left[p_t^* + \beta \pi_{t+j} \right] \tag{3}
\]

The adjusted price level of firms is affected by the desired price level, \( p_t^* \) and also influenced by the lagged inflation \( \pi_{t-1} \), which presents the bounded rationality of firms. The parameter \( \beta \) represents the degree of bounded rationality, which means the relative degree to the lagged inflation relevant to the desirable price level, and satisfies \( \beta > 0 \); then, the ratio of \( \pi_{t-1} \) to \( p_t^* \) is \( \frac{1}{1 + \beta} \cdot \frac{\beta}{1 + \beta} \). The right side of the second equal sign is one that is solved forward iteratively.
This specification is the simplest one for bounded rationality. We can also assume one that it includes the anchor effect, such as is assumed in Ball and Mazumdar (2011), to express bounded rationality of firms:

\[ x_t = \rho \left( p_t^* + \beta \pi_{t-1} + \vartheta A_t \right) + (1 - \rho) E_{t-1} x_{t+1} \]

where \( A_t \) explains the anchor effect. The ratios of \( p_t^* \) to \( \pi_{t-1} \) to \( A_t \) then become \( \frac{1}{1 + \beta + \vartheta} \cdot \frac{\beta}{1 + \beta + \vartheta} \cdot \frac{\vartheta}{1 + \beta + \vartheta} \), respectively. This specification, however, does not affect any qualitative conclusions. For simplicity, we maintain the simple specification as Equation (3).

With some tedious algebra, which can be found in detail in Appendix A, we yield the following equation for inflation:

\[ \pi_t = E_{t-1} \pi_{t+1} + \frac{\alpha \rho^2}{1 - \rho} y_t + \frac{\beta \rho^2}{1 - \rho} \pi_{t-1} \]  

\[ \text{(4)} \]

We call this equation the sticky-price with bounded rationality Phillips curve (SPBR). The actual inflation depends not only on the actual output and inflation rate expectation, but also on the lagged inflation, a hybrid new Keynesian Phillips curve. The coefficient of lagged inflation depends on the frequency of price adjustment, \( \rho \) and the degree of bounded rationality \( \beta \). The coefficient of output gap depends on \( \rho \) and the degree of output gap on desired price level \( \alpha \). Repeatedly, we obtain the SPBR deriving from two underlying assumptions, the sticky-price mechanism and the price-setting with bounded rationality. These assumptions imply that both sides of the market and firms are imperfect.

3. Simulation and Estimation

We have presented the SPBR, and will now examine its dynamic...
properties. To achieve this and present a comparison with previous studies, we first need to introduce a hybrid Phillips curve differing from Equation (4), as follows:

$$\pi_t = \phi_f E_t \pi_{t+1} + \phi_b \pi_{t-1} + \lambda y_t$$ (5)

where $\lambda$ is a coefficient for output gap, and can be regarded as $\frac{\omega}{1-\rho}$ related to Equation (4); and parameter $\phi_f$ and $\phi_b$ represent the contributive degree of expected and lagged inflation to the actual one, respectively. Although Equation (4) and Equation (5) are not strictly the same, using the parameters in Equation (4), we can regard the relative ratio of the two parameters as: $\frac{1}{1+\omega} \cdot \frac{\omega}{1+\omega} = \frac{\omega}{1-\rho}$ where $\omega = \frac{\beta}{1-\rho}$.

To complete the model, we need to adopt an expectational IS curve of a hybrid form, as follows:

$$y_t = \phi_f E_t y_{t+1} + \phi_b y_{t-1} - \delta(i_t - E_t \pi_{t+1})$$ (6)

where $i_t$ is the nominal interest rate; $\delta$ is the inverse of the degree of relative risk aversion; and $\phi_f$ and $\phi_b$ are the parameters, satisfying $0 < \phi_f < 1$ and $0 < \phi_b < 1$. Equation (6) can be obtained from the Euler’s equation using a habit formation utility function.

We select $\delta = 1.5$ and $\alpha = 0.1$, as usual for simulation, to display the impulse responses of inflation and output dynamics to a particular shock. We select $\phi_f = \phi_b = 0.5$ and $\phi_f = \phi_b = 0.5$ for two reasons. First, the sum of parameters $\phi_f$ and $\phi_b$ are set equal to 1, as presented by previous studies.

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4 Equation (3) is almost the same as Equation (4). Beginning with a slightly different setup, we can get the equation easily (e. g., Walsh, 2005, Chapter 8).
Second, following our GMM estimation as described next, the values of $\phi_f$ and $\phi_b$ are about 0.5. Although many previous studies, such as Gali and Gertler (1999), the pioneer study using GMM, estimate that the forward-looking coefficient, $\phi_f$, is larger than the backward-looking one, $\phi_b$, and the values of $\phi_f$ and $\phi_b$ are about 0.6 and 0.4, respectively, the selection of instrument variables in their models is arbitrary. We will discuss this topic in detail next and in Appendix B. Moreover, Kurmann (2007), using the Maximum-Likelihood (ML) approach, obtains an estimation result that the value of $\phi_f$ is 0.542 using same data set as Gali and Gertler (1999). Jondeau and Bihan (2005) report that the values of $\phi_f$ of the U.S. are 0.480 with output gap and 0.525 with real ULC; those of the Euro area are 0.496 and 0.540, respectively.

The model including Equation (5) and (6) implies that as long as the central bank is able to affect the real interest rate through its control of the nominal interest rate, monetary policy can affect real output. The changes in real interest rate alter the optimal time path of consumption. Firstly, we consider a sudden drop in the level of aggregate demand. Fig. 1 illustrates the impact of a monetary policy shock, a decrease in the nominal interest rate $i_t$ in the model; policy rule agrees with the Taylor rule as follows:

$$i_t = q_1\pi_t + q_2y_t + \upsilon_t$$

The parameter values are $q_1 = 0.5$ and $q_2 = 1.5$, as usual. In addition, the policy shock, $\upsilon_t$ in the policy rule is assumed to follow an AR(1) process given by $\upsilon_t = \xi \upsilon_{t-1} + \epsilon_t$, with $\xi = 0.8$. The impulse response that we select is that the nominal interest rate; $i_t$ unexpectedly falls by 10 percent at time zero.

Fig. 1 shows that the apparent fall in the nominal interest rate causes
inflation and the output gap to rise immediately, and then gradually dissipate
over time. The difference between our model and the new Keynesian model
is when we examine the response of inflation. It is well-known that the
maximum impact of a rise in inflation occurs immediately in the new
Keynesian model; whereas, the maximum impact occurs at five quarters in
our model. Thus, inflation persistence could be well described. The output
gap rises immediately and converges to zero over time, although it falls to a
slight minus.

We then consider a sudden shock in the money supply. Fig. 2 shows
inflation and the output gap dynamic with monetary rule as follows:

\[ i_t = \frac{1}{\sigma} y_t + \mu m_t ; \]

\[ \Delta M_{t+1} = \tau \Delta M_t + \omega_t ; \]

\[ m_t = m_{t+1} - \Delta M_{t+1} + \pi_{t+1} \]

where \( M_t - p_t = m_t \). The policy shock is assumed to follow an AR(1)
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process. The parameter values are; $\mu = 2$, $\delta = 1.5$ and $\tau = 0.8$. The impulse response that we select is that the nominal money supply; $\Delta M_t$ unexpectedly increases by 10 percent at time zero.

The quantitative easing policy in the money supply brings about the output gap and growth of inflation, which gradually dissipates over time. As same as the case of Taylor rule, the difference between our model and the new Keynesian model is apparent when the response of the inflation rate is examined. Contrary to the new Keynesian model that raises the maximum impact immediately, the maximum impacts on inflation and output gap occur at five quarters in our model.

Inflation persistence has been well described in our model. Although the maximum impact on inflation occurs five quarters after shock, which is slightly quicker than the actual data (Fuhrer and Moore, 1995), it is easy to change the peak period at and after eight quarters by a simple adjustment within the reasonable parameters range.

We now present and discuss the GMM estimate of the hybrid models described above. Using the GMM technique, we report parameters in Table 1.
Table 1
Estimation of the Hybrid Philips Curve by GMM

<table>
<thead>
<tr>
<th>Case</th>
<th>Region</th>
<th>Period</th>
<th>$\phi_b$</th>
<th>$\phi_j$</th>
<th>$\lambda_{ULC}$</th>
<th>$\lambda_{GDP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>U.S.</td>
<td>(1960:Q1-1997:Q4)</td>
<td>0.355</td>
<td>0.627</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.039</td>
<td>0.023</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ULC</td>
<td></td>
<td>0.323</td>
<td>0.685</td>
<td></td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.047</td>
<td>0.051</td>
<td></td>
<td>(0.003)</td>
</tr>
<tr>
<td>II</td>
<td>US</td>
<td>(1960:Q1-2013:Q2)</td>
<td>0.505</td>
<td>0.515</td>
<td></td>
<td>-0.007</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.015</td>
<td>0.018</td>
<td></td>
<td>(0.008)</td>
</tr>
<tr>
<td></td>
<td>ULC</td>
<td></td>
<td>0.518</td>
<td>0.498</td>
<td></td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.018</td>
<td>0.016</td>
<td></td>
<td>(0.010)</td>
</tr>
<tr>
<td>III</td>
<td>Euro area</td>
<td>(1970:Q1-1999:Q4)</td>
<td>0.387</td>
<td>0.608</td>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.059</td>
<td>0.059</td>
<td></td>
<td>(0.005)</td>
</tr>
<tr>
<td></td>
<td>ULC</td>
<td></td>
<td>0.367</td>
<td>0.628</td>
<td></td>
<td>0.121</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.091</td>
<td>0.091</td>
<td></td>
<td>(0.094)</td>
</tr>
</tbody>
</table>

Note: in all cases, the dependent variable is quarter inflation measured using the GDP Deflator. Standard errors are shown in brackets. Case I is a result of the U.S. from 1960:Q1 to 1997:Q4, cited by Gali, Gertler and Lopez-Salido (2005)'s Table 1. Case III is a result of the Euro area from 1970:Q1 to 1999:Q4, cited by Jondeau and Le Bihan (2005)'s Table 1. In Case II, the instrument set includes a constant term, one lag and lead inflation rate, and current real UCL or output gap.

which estimate the hybrid new Keynesian Phillips curve as follows:

$$\pi_t = \phi_b \cdot \pi_{t-1} + \phi_j \cdot \pi_{t+1} + \lambda_{ULC} \cdot ULC_t,$$

$$\pi_t = \phi_b \cdot \pi_{t-1} + \phi_j \cdot \pi_{t+1} + \lambda_{GDP} \cdot y_t,$$

where $\lambda_{ULC}$ is a parameter when the explanatory variable is a real Unit Labor Cost (ULC); and $\lambda_{GDP}$ is another parameter when the explanatory variable is the output gap. Standard errors are shown in brackets. Case I is a result of the U.S. from 1960:Q1 to 1997:Q4, cited by Gali, Gertler and Lopez-Salido.
The data that we use (Case II) is quarterly for the U.S. over the period of 1960:Q1 to 2013:Q2, drawn from OECD Business Sector Data for the U.S. The inflation rate is the percent change of inflation measured using the GDP Deflator. The output gap is the percent deviation of real GDP from its trend, computed using the Hodrick-Prescott filter. The instrument set that we select includes a constant term, one lagged and lead inflation, and the actual UCL or output gap; whereas, Gali, Gertler and Lopez-Salido (2005) select an instrument set that includes two lags of detrended output, real marginal costs and wage inflation and four lags of inflation. Jondeau and Le Bihan (2005) instrument set is a similar one. As Muth (1961) emphasized, the variance of predicting error needs to be minimal. Since increasing extra lagged or lead variables in the instrument set causes the variance to be bigger, which violates rational expectations, we select an instrument set which is the simplest one as above to keep its variance to a minimum. Appendix B gives the theoretical explanation in detail.

Firstly, as Table 1 shows, the parameters of \( \phi_b \) and \( \phi_f \) are all statistically significant with both real ULC and output gap in three case. This indicates that the backward-looking inflation is important to the actual one, as well as to the forward-looking one. Secondly, in each case, the estimates of \( \phi_b \) and \( \phi_f \), obtained by real ULC or output gap, are very close. Thirdly, except for Case I with real ULC, the parameters of real UCL and output gap, \( \lambda_{ULC} \) and \( \lambda_{GDP} \), are not statistically significant; moreover, the value of these parameters, compared with \( \phi_b \) and \( \phi_f \), are very small. This evidence suggests that the choice of forcing variable hardly influences the actual inflation, which contradicts the claim by Gali and Gertler (1999) (Jondeau and Le Bihan, 2005).

In Case II, we estimate that the value of \( \phi_b \) and \( \phi_f \) are around 0.5.
Comparing these values with those of Case I and III, the expected estimated length, the differences arise from the setup of the instrument set as mentioned above. Interestingly, using the ML approach, Jondeau and Le Bihan (2005) report that the estimates of $\phi_b$ and $\phi_f$ for both the U.S. and Euro area are very close to 0.5. Kurmann (2007) reports parallel estimates. This evidence implies that the importance of the lagged inflation to the current one is the same as that of the expected one. This contradicts the claim by Gali and Gertler (1999), who state that the role of forward-looking inflation is more important than that of the backward-looking one in the formation of inflation expectations. Considering all of the evidence above, we believe that the value 0.5 of $\phi_b$ and $\phi_f$ are feasible values.

The estimated result of Blanchard and Gali (2007) is worthy of being mentioned. They estimate that the coefficients of backward-looking inflation, forward-looking inflation and unemployment rate are 0.66, 0.42 and -0.20, respectively, with statistical significance. Although it is very remarkable to obtain a large enough unemployment rate coefficient with statistical significance, they use annual U.S. data on inflation and a four lags instrument set of variables. Having several estimations with quarterly data of inflation and unemployment rate consistent with Blanchard and Gali (2007), we unfortunately cannot obtain a meaningful coefficient of unemployment rate.

To sum up these estimation results, the data and instrument set used in empirical analysis have to be selected with extreme caution. On the other hand, the estimation results, including our model, are far from satisfactory, which suggests that theoretical and empirical analysis in this field must be drastically improved.

4. Discussion

4.1 Sticky Price versus Bounded Rationality

We have presented the SPBR model, and have simulated and estimated its dynamic properties. Now we return to theoretical considerations, and
again investigate how the lagged and expected inflation affect the actual one in our model, or similarly, how sticky price and bounded rationality affect inflation.

Recalling Equation (3), it is easy to verify that Equation (4) will become an NKPC form when \( \beta = 0 \). Define that \( \frac{\beta \rho^2}{1 - \rho} = D \) and \( D > 0 \), and then we can obtain \( \frac{\partial \beta}{\partial \rho} < 0 \) \(^5\) in the case that \( D \) is constant. Since the parameters \( \rho \) and \( \beta \) signify the degrees of sticky price and bounded rationality, the evidence indicates that the more stickiness is in the price change, the more is the degree of bounded rationality in the price setting, and vice versa. It is easy to understand this relation intuitively. It is intelligible when a simple numerical example is shown. In the special case of \( D = 1 \), which corresponds to the case of \( \phi_b = \phi_f = 0.5 \) as above, when \( \rho = 0.25 \), then \( \beta = 12 \); when \( \rho = 0.5 \), then \( \beta = 2 \); and when \( \rho = 0.67 \), then \( \beta = 0.75 \). When \( \rho \) is small, i.e., the probability in any given period that the firm can adjust its price during that period is small, the degree of bounded rationality, \( \beta \) becomes big, the firm will put more weight on the ratio on the lagged inflation rather than on the desirable price level in the price setting. The share on the desirable price level is 7.7 percent, and the share on the lagged inflation is 92.3 percent when \( \beta = 12 \); when \( \beta = 2 \), the share on lagged inflation becomes 66.7 percent. Considering that the coefficient of the lagged inflation term is around 1, when estimating the NKPC, the value of \( \beta \) is not too big to be permissible.

4.2 The Source of SPBR

It is well known that since the lagged term of inflation exists, the delay in

\[ \frac{\beta \rho^2}{1 - \rho} = D \quad \text{and} \quad D > 0 \quad \text{we can get} \quad \beta = \frac{D (1 - \rho)}{\rho^2} \quad \text{and then} \]

\[ \frac{\partial \beta}{\partial \rho} = - \frac{D \rho^2 - 2 D (1 - \rho) \rho}{\rho^4} < 0 \]

\(^5\) Since \( \frac{\beta \rho^2}{1 - \rho} = D \quad \text{and} \quad D > 0 \quad \text{we can get} \quad \beta = \frac{D (1 - \rho)}{\rho^2} \quad \text{and then} \]
inflation adjustment arises, and the maximum impact on inflation will occur several quarters later, which can avoid the criticism of Ball (1994). Moreover, as many previous studies pointed out, such as Gali and Gertler (1999), the hybrid new Keynesian Phillips curve, compared with the NKPC, exhibits good performance in empirical analysis.

When setting the price level, a firm must be concerned with the desirable price level and an expected inflation in the NKPC model. In the SPBR model, the firm is also concerned about the lagged inflation and other factors, such as the anchor effect, which are not set in the model. The difference between the two models is whether or not the bounded rationality of the firm is admitted. In the case without bounded rationality in the price setting, the firm is assumed to be a market-clearing one, which performs profit maximization and evaluates the degree of nominal rigidity in the future completely. In the case with bounded rationality, however, the firm is assumed to be not a perfect market-clearing one, which also performs profit maximization and evaluates the degree of nominal rigidity; at the same time, the firm is also assumed to refer to lagged inflation, which is the easily obtainable and inexpensive information that the firm can hold.

Taking lagged inflation in the model, we can obtain the SPBR, a form of the hybrid new Keynesian Phillips curve. The essence of the SPBR that we want to emphasize is, as well as the lagged inflation in expecting the actual inflation, the importance of the bounded rationality of the firm on inflation dynamics. The bounded rationality of the firm, accompanying sticky price, will rebalance the model without one-sided imperfection.

5. Conclusion

A great deal of advancement in theoretical and empirical analysis in both price setting and sticky inflation has been achieved in the past two decades. However, a number of questions have remained unanswered. A model that

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6 That the firm cannot alter the price by sticky price results from the assumption that the market is assumed to be imperfect.
include the lagged terms, which is called the hybrid new Keynesian model and short for microeconomic foundation, is being used extensively due to its good performance on actual data. However, we realized that there is an unbalanced specification in the standard new Keynesian model. In the model, stickiness is assumed in the price setting, and then an individual firm has a fixed probability to be able to change its price in any given period, which means that the market is imperfect. On the other hand, an individual firm is assumed to be one, which can conduct profit maximization and calculate the degree of nominal rigidity in the future completely.

In order to avoid this unbalanced specification and obtain a model which is suitable to the actual data, we suppose that firms choose the price with bounded rationality. Concretely, we assume that firms refer to lagged inflation in the price setting, one of the simplest forms to express bounded rationality; thus, we obtain the hybrid new Keynesian Phillips curve to express inflation dynamics, which we call the sticky price with bounded rationality Phillips curve.

As previous studies proved with the hybrid new Keynesian model, the SPBR possesses a remarkable feature compared with the NKPC, i.e., inflation persistence exists well. There is an interesting relation between sticky price and bounded rationality, in which the more stickiness that is in the price changing, the more is the degree of bounded rationality in the price setting, and vice versa. This is easy to understand intuitively. Furthermore, firms will place most weight of the ratio on the lagged inflation rather than on the desirable price level, in the price setting.

Taking in the bounded rationality or its concrete form, lagged inflation in the model, we obtain the SPBR. The essence of the SPBR that we want to emphasize is, as well as the lagged inflation in expecting the actual inflation, the importance of the bounded rationality of the firm on inflation dynamics.

References

Angeloni, I., Aucremanne, L., Ehrmann, M., Galf, J., Levin, A., Smets, F.


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Appendix A

Transforming Equation (2), we obtain the adjustment price level at time $t$ and time $t+1$ as follows:

$$x_t = \frac{1}{\rho} p_t - \frac{1 - \rho}{\rho} p_{t-1}$$

$$x_{t+1} = \frac{1}{\rho} p_{t+1} - \frac{1 - \rho}{\rho} p_t$$

Replacing $p_t^*$ of Equation (1), $x_t$ of Equation (2') and $x_{t+1}$ of Equation (2'') into Equation (3), we can obtain Equation (4):

$$\frac{1}{\rho} p_t - \frac{1 - \rho}{\rho} p_{t-1} = \rho p_t + \alpha \rho \pi_t + \beta \rho \pi_{t-1} + \frac{1 - \rho}{\rho} E_t p_{t+1} - \frac{(1 - \rho)^2}{\rho} - p_t$$

$$\left[ \frac{1}{\rho} - \rho + \frac{(1 - \rho)^2}{\rho} \right] p_t - \frac{1 - \rho}{\rho} p_{t-1} = \alpha \rho \pi_t + \beta \rho \pi_{t-1} + \frac{1 - \rho}{\rho} E_t p_{t+1}$$
\[
\frac{1 - \rho}{\rho} p_t - \frac{1 - \rho}{\rho} p_{t-1} = \rho \alpha y_t + \beta \rho \pi_{t-1} + \frac{1 - \rho}{\rho} E_t p_{t+1} - \frac{1 - \rho}{\rho} p_t
\]

\[
\frac{1 - \rho}{\rho} \pi_t = \frac{1 - \rho}{\rho} E_t \pi_{t+1} + \alpha \rho y_t + \beta \rho \pi_{t-1}
\]

\[
\pi_t = E_t \pi_{t+1} + \frac{\alpha \rho}{1 - \rho} y_t + \frac{\beta \rho}{1 - \rho} \pi_{t-1}
\]

Appendix B

According to Muth (1961), the rational expectations hypothesis means that the expected inflation rate \( \pi_t^e \) is an unbiased predictor of the actual inflation rate \( \pi_t \), that is:

\[
\pi_t = \pi_t^e + \nu_t, \quad E_t \nu_t = 0, \quad E \nu_t = 0
\]

Assuming that \( \nu_t \) is independently and identically distributed normal with variance \( \sigma^2 \), we know that

\[
\pi_t = \pi_{t-1} + \nu_{t-1}
\]

where \( \nu_{t-1} \) is a realized predicting error. Using two periods moving average instead of \( \pi_t \) in the first equation, we get another predictor \( \pi_t^e \) below:

\[
\frac{\pi_t + \pi_{t-1}}{2} = \pi_{t-1} + \frac{\nu_{t-1}}{2} = \pi_t^e + \nu_t
\]

The variance of predictor \( \pi_t^e \) is larger than that of \( \pi_t^e \).

\[
E(\pi_t^e - \pi_{t-1})^2 = E\left(\nu_t - \frac{\nu_{t-1}}{2}\right)^2 = E\left(\nu_t^2 + \frac{\nu_{t-1}^2}{4} - \nu_t \nu_{t-1}\right) = \frac{5}{4} \sigma^2 > E(\pi_t^e - \pi_{t-1})^2 = \sigma^2
\]
Note that $E \nu_t \nu_{t-1} = 0$. It can be presumed easily that the more lag or lead terms, the larger the variance of the predictor. This result is very appealing, and will hold the generality when the distribution term bears the reproductive property. For this reason, we select an instrument set only including one lag and lead inflation rate to keep the variance of the predicting error to a minimum.